[Kim, 3(7): July 2016] ISSN 2348 – 8034
DOI-10.5281/zenodo.59145 Impact Factor- 4.022

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

A STUDY ON THE EIGEN MODES OF PCF VARYING THE POSITION OF THE DIELECTRIC HOLES BY FEM

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ABSTRACT

FEM has been used to obtain the eigen-modes of PCF involving the varied distribution of the dielectric holes. PCF was assumed to be constructed with the background material of high-index dielectric and the several circular holes of low-index one. These holes were the dielectric such like the air and surrounded the central region of PCF. FEM has given the results that the spectra were varied according to the position and dimension of the dielectric air holes. Each eigen-modes have been identified based on the spectra of the conventional optical fiber. The spectra have shown that the background material restricted the distribution of electromagnetic field to the central region. From these results, it has reconfirmed that the air dielectric holes combined with background material to confine the electromagnetic field into the small central region of PCF.

Keywords: *FEM, PCF, eigen-mode, dielectric, waveguide.*

I. INTRODUCTION

From several decades ago, PCF(Photonic Crystal Fiber) has attracted a considerable amount of attention from the fiber communication field[1]. PCF has been known as a suitable candidate for many applications in this field because of its processing dispersion, coupling and nonlinear properties. PCF with a periodic transverse microstructure have made it possible to realize low-loss waveguides in the practice[2]. The most prominent feature of PCF may be ability to concentrate the light on a small area of the propagating cross section. Light confinement has been understood by two different mechanisms: the index-guiding, and photonic band-gap effects[3]. Among these, the eigen-mode reflecting the physical properties of PCF can be easily understood through the index-guiding effect. But the fabrication of PCF usually involves costly facilities and time-consuming procedures.

Modeling tools are indispensable in research and development related to the fiber communication properties of PCF. Modeling tools are required to characterize and design the structures/devices before their realization. FEM(Finite Element Method) has been known as one of the most important method to understand the eigen-properties of PCF. FEM could be applied to the system of the geometrically complex structure. It may also easily involve the boundary condition in the process of calculation. In this study, FEM has been applied to calculate the eigen-modes of PCF. The space of PCF was divided into the linear triangular mesh. The eigen-equation was constructed with tangential edge vectors of the element triangular mesh. The eigen-modes were obtained by using Krylov-Schur iteration method [4]. TE(Transverse Electric) and TM(Transverse magnetic) modes could be calculated by giving different boundary conditions on the surface of the waveguide.

PCF model was constructed with background material of high-index dielectric and the several circular holes of low-index one. The circular holes were regularly positioned in PCF of square shape. The holes substituted on the background material but did not changed the structure of PCF totally. The holes were added from the circumference to the center of PCF sequentially. As a final result, the spectra of the eigen-modes have illustrated in schematic representation. The eigen-modes were understood based on the spectra of the conventional waveguide which did not contained any dielectric holes.

II. FINITE ELEMENT FORMULATION

Previously, we have studied on the eigen-modes of square waveguide made with two different dielectric[5]. In there, we have illustrated the process of constructing the eigen-equation from FEM and calculating the eigen-modes through Krylov-Schur iteration method. The same method has been used to calculate the eigen-modes of PCF in this study. The established eigen-modes have been obtained by solving the fallowing vector Helmholtz equation

ISSN 2348 – 8034
Impact Factor- 4.022

$$\vec{\nabla}_t \times \left(\frac{1}{\nu}\vec{\nabla}_t \times \vec{F}_t\right) - k^2\zeta\vec{F}_t = 0 \qquad (1)$$

where $k = \omega\sqrt{\epsilon_o\mu_o}$ is the propagation wave number and, for the TE mode $\vec{F}_t = \vec{E}_t$ (transverse electric field strength), $\nu = \mu_r$(relative permeability μ/μ_o), $\zeta = \epsilon_r$ (relative permittivity ϵ/ϵ_o) and, for the TM mode $\vec{F}_t = \vec{H}_t$(transverse magnetic field strength), $\nu = \epsilon_r$, $\zeta = \mu_r$ respectively[6]. The eigen-equation has been obtained from FEM. It has been carried out using the Galerkin method of weighted residuals to construct the linear equation. The shape function has been constructed with the constant tangential edge vectors of the triangular element[7]. Triangle barycenter coordinate are related to the vertex of triangle

$$\begin{pmatrix}\mathcal{L}_1\\ \mathcal{L}_2\\ \mathcal{L}_3\end{pmatrix} = \frac{1}{2A}\begin{pmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix} \qquad (2)$$

where $a_i = x_j y_k - x_k y_j, b_i = y_j - y_k, c_i = x_k - x_j$ $(i, j\ and\ k\ are\ cyclic\ vetix\ indices\ )$ and A is an area of the triangular element mesh. With these coordinates, the tangential edge vectors $\vec{W}_{ti}$ are made

$$\vec{W}_{ti} = l_i(\mathcal{L}_j\vec{\nabla}_t\mathcal{L}_k - \mathcal{L}_k\vec{\nabla}_t\mathcal{L}_j) \qquad (3)$$

Where l_i is a length of the edge opposite to the vertex i and i, j, k are indices of the barycenter coordinates in cyclic ordering. The field strength in a single triangular element can be calculated with these tangential edge vectors,

$$\vec{F}_t = \sum_{i=1}^{i=3} e_{ti}\vec{W}_{ti} \qquad (4)$$

where e_{ti} are coefficients to be found in the diagonalizing processes of the eigen-equation. Using this function the Helmholtz equation for a triangular element can be made as following

$$[S_{el}]\{F_t\} = k^2[T_{el}]\{F_t\} \qquad (5)$$

where $[S_{el}] = \frac{1}{\nu}\iint_A (\vec{\nabla}_t \times \vec{W}_{tm}) \cdot (\vec{\nabla}_t \times \vec{W}_{tn})ds$ and $[T_{el}] = \zeta \iint_A \vec{W}_{tm} \cdot \vec{W}_{tn}ds$.

Subsequently, these element matrix equations can be assembled over all triangular meshes of the waveguides to obtain a global eigen-matrix equation.

$$[S]\{F_t\} = k^2[T]\{F_t\} \qquad (6)$$

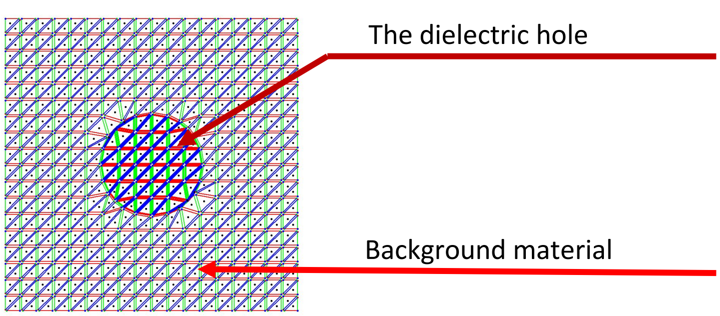
In this equation, $[S]$ and $[T]$ are $n \times n$ square matrices and $\{e_t\}$ is $n \times 1$ column matrix where n is a total number of the edges composing the mesh of the waveguides. When obtaining the eigen-modes of the TM mode, the tangential components of the electric fields must satisfy the Dirichlet boundary condition. This is accomplished in the course of implementing the programs by cancelling the edge components of the triangular elements which coincide with the wall of the waveguides.

As mentioned in the previous study, it has been well known that the Krylov-Schur iteration method has been known as the most reliable technique for finding the prominent eigen-modes[8][9]. So this iteration method was applied to finding the eigen-modes of PCF in the study. The method was more efficiently implemented in finding specific eigen-modes by performing the shift-invert strategy as following

$$\lambda_o\{F\} = \frac{[T]}{[S]-\sigma[T]}\{F\} = [M]\{F\} \qquad (7)$$

where $\lambda_o = \frac{1}{k^2-\sigma}$. The sparsity and symmetry of the eigen-equation might be lost, but by this strategy the convergent rate is more promoted at the specific value σ. Subsequently, the Krylov-Schur iteration method has been performed on this square matrix [M].

III. RESULT & DISCUSSION



PCF with a dielectric mesh

DOI-10.5281/zenodo.59145 Impact Factor- 4.022

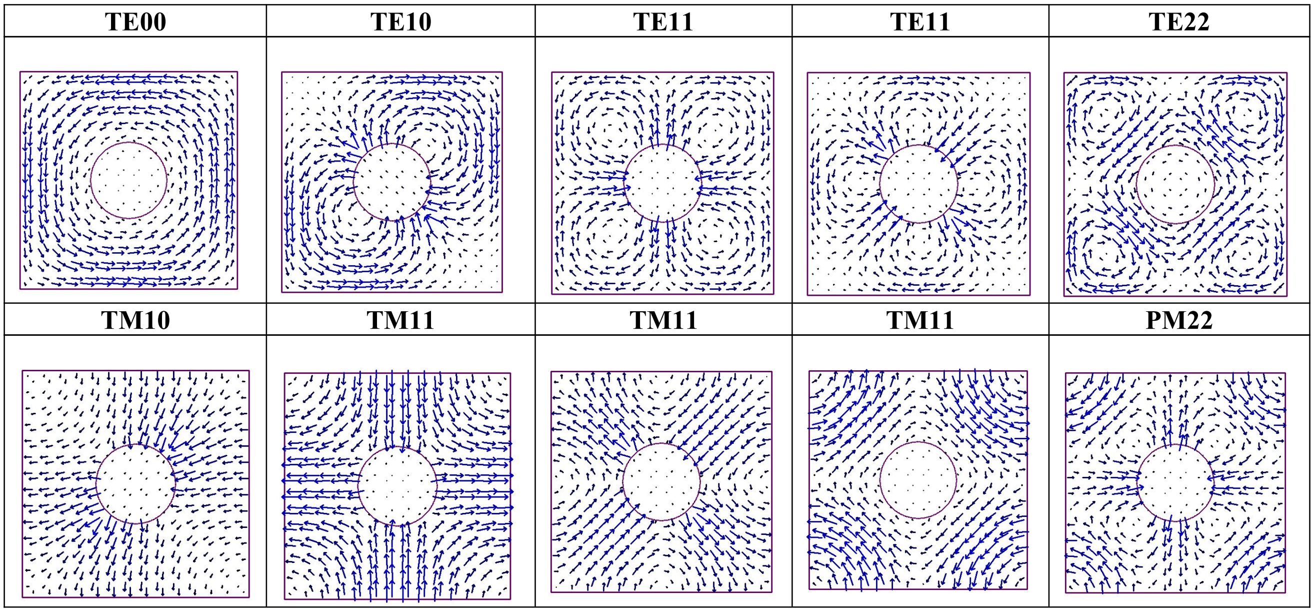


Figure 1. The spectra of PCF with a dielectric hole at the central position.

In this study, FEM was carried out to investigate the eigen-properties of PCF depending on the various distribution of the dielectric holes. PCF was assumed to be constructed with the background material of high-index dielectric and the several circular holes of low-index one. The relative dielectric constant of the background material and the circular holes were assumed to be 13.0 and 1.0 respectively. As in the previous study, the lateral surface of the waveguides were assumed to be perfect conductor. The reason for it was that this assumption is convenient to obtain TM modes by ignoring the variables on the surface. The cross section of PCF was the square form and the same size without differentiating the distribution of dielectric holes. The dielectric holes were positioned around the center of PCF and distributed outwardly from there. Typical PCF with a dielectric hole is represented in fig. 1. To understand the eigen-properties of PCF, the eigen-equation was established by using FEM. The cross section of PCF was restructured by the triangular mesh. The vector Helmholtz eq. (1) for a triangular element mesh was interpreted into the matrix eq. (5). These matrices equation were assembled over the entire region according to the global edge numbering to obtain a global matrix equation as like eq. (8). The eigen-modes were calculated by performing the Krylov-Schur iteration on the square matrix [M] of eq. (8). The eigen-modes were the column vectors of the similar transforming matrix which convert the square matrix [M] to a Shure form. The similar transforming matrix was where n was the edge number of the triangular mesh and 20 was the arbitrary determined dimension of the Arnordi contracted matrix. The Shure form was the upper diagonal matrix with 20x20 dimension.

ISSN 2348 – 8034
Impact Factor- 4.022

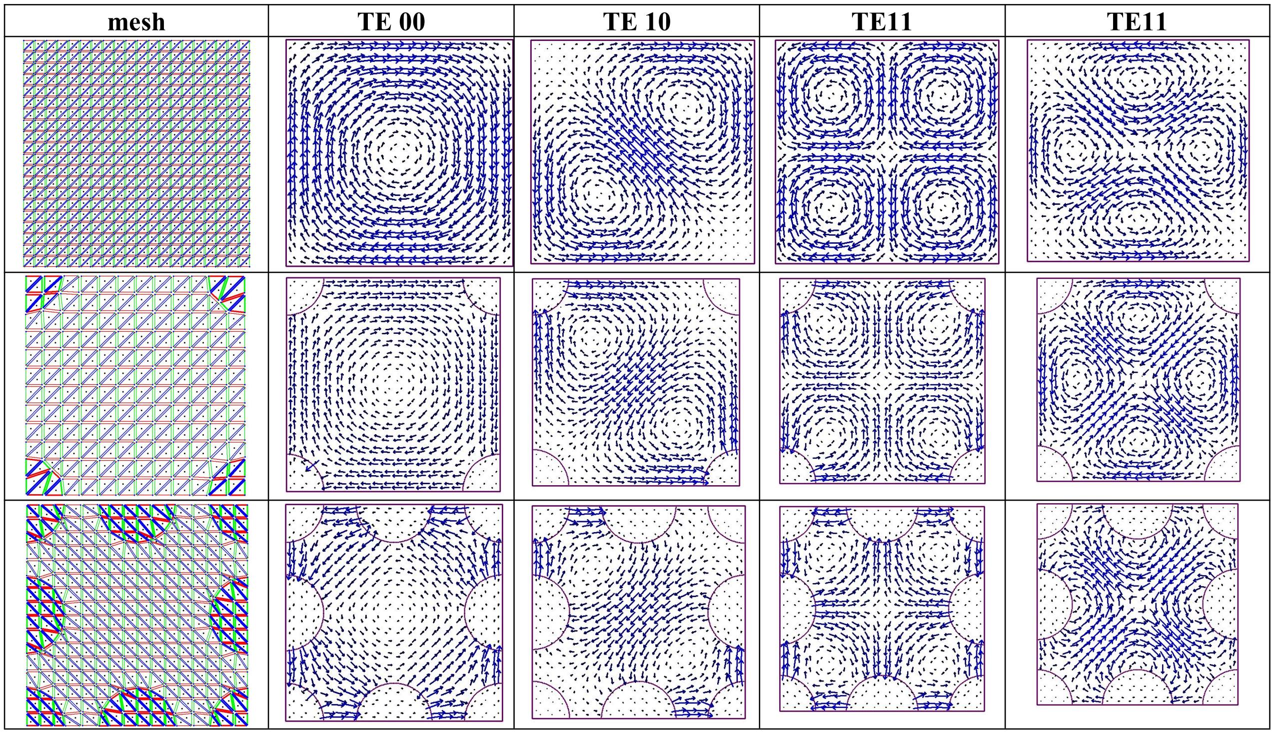


Figure 2. Schematic representation of the TE mode spectra.

The Following description are about the spectra of the TE and TM modes respectively resulted from the Krylov-Schur iteration. Firstly, fig.1 represent the spectra of the simple PCF structure which contain a dielectric hole at the central position. The spectra are similar to the eigen-mode of the square waveguide without the dielectric holes illustrated in first rows of fig. 2, 3 Comparing between these spectra, it could be identified that the spectra are different each other only at the position of the dielectric hole. Fig.1 have shown the electromagnetic field that the dielectric hole exclude perfectly from the central position of PCF. This feature suggested that by controlling the spatial arrangement of the dielectric holes in PCF the electromagnetic wave could be limited into a restrict area. Basing on this idea, the dielectric holes were positioned around the central position to confined eigen-mode in there. Fig.2, 3 represent the spectra of TE and TM modes respectively according to addition of the dielectric holes to PCF. The dielectric holes have been added from the circumference to the central region of PCF sequentially. Fig. 2, 3 are the spectra plotted against to adding the dielectric holes to the background material. The spectra of the first row are the eigen-modes without the dielectric holes. The spectra following to them were resulted from PCF containing the dielectric holes. These were understood basing on the interaction between the dielectric holes and the background material. By Snell's law, the electromagnetic field might be confined into the region occupied by the high dielectric material. As can be seen in the spectra, this phenomenon was more intensified as the dielectric holes increased.

mesh	TM 10	TM 11	TM11	TM33

DOI-10.5281/zenodo.59145
Impact Factor- 4.022

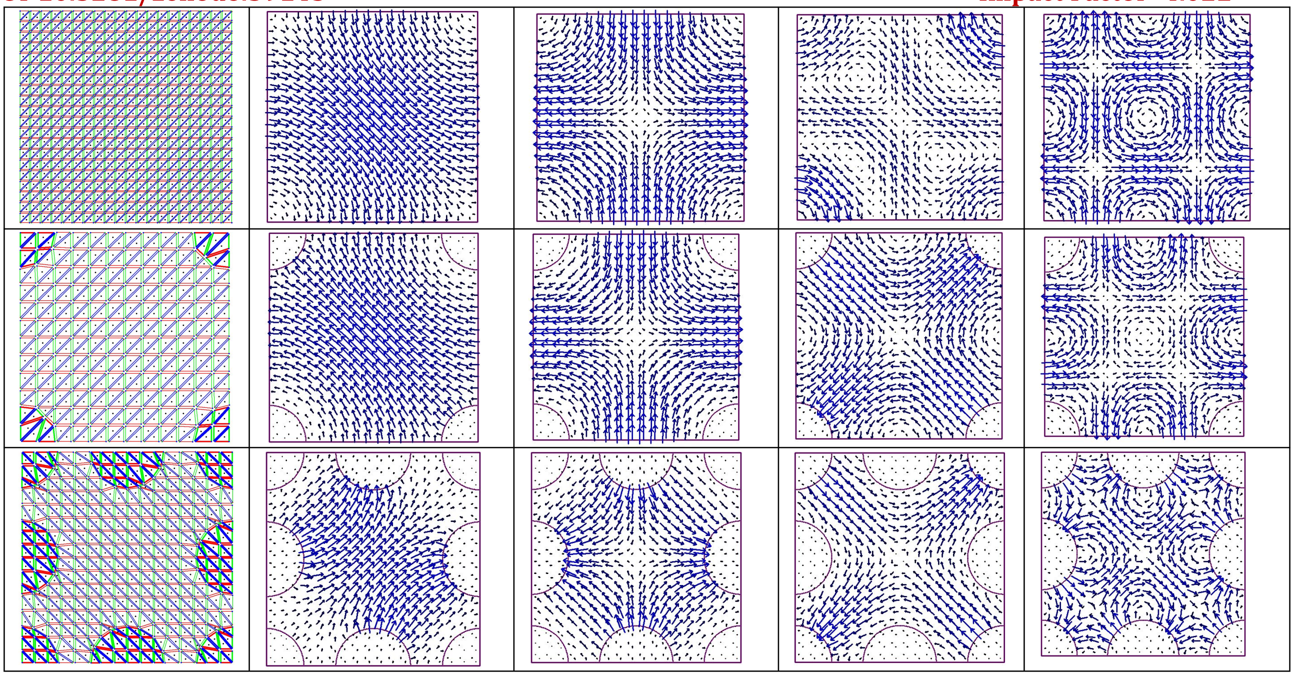


Figure 3. Schematic representation of the TM mode spectra.

Fig. 4 represent the spectra of the eigen-modes resulted from the PCF which contain the dielectric holes surround the central region. At the final stage, the electromagnetic field was confined perfectly in there without any outward leakage. Without differentiating TE and TM modes, the last spectra were revealed the similar feature. They were appeared in the small central region, but similar to the spectra without the dielectric holes. These spectra did not depended on the boundary condition. There were TE and TM mode spectra simultaneously in the result of one calculation. Perfect conducting boundary condition of PCF surface did not affect the eigen-mode. The cause has been inferred from the shielding effect of the dielectric holes against to the boundary condition of the waveguide surface

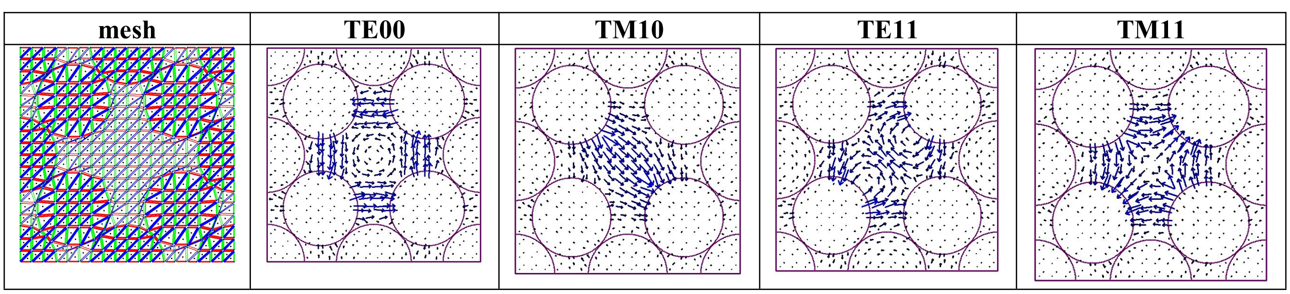


Figure 4. Schematic representation of the perfectly confined eiegen-mode spectra.

The dielectric holes surrounding the central region seemed to be a barrier restricting the electromagnetic field to this small region. This property has meant that PCF did not leak the energy of the propagating electromagnetic wave. The dielectric holes did not permit the electromagnetic wave to overcome the finite central region. These properties were advantages that could not be expected from the conventional waveguide. PCF with such characteristics have been utilized in the communication field requiring a high density and a precise signal [10].

ISSN 2348 – 8034
Impact Factor- 4.022

IV. CONCLUSION

FEM has been carried out to investigate the eigen-modes of PCF. Krylov-Schur iteration method was applied to calculate the eigen-modes of TE and TM. The schematic representation for these spectra have shown that the electromagnetic wave is concentrated in the core region of the greater dielectric constant. From the spectra, it has been identified that the dielectric holes restrict the electric field in the central region of PCF and determine the eigen-modes in there. From this result, it can be certified that PCF may be confine and concentrate the electromagnetic wave in the small area by varying the dielectric holes.

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